Modeling Firms

Short Course on CGE Modeling, United Nations ESCAP

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July 22-26, 2013

Introduction

- Now that we understand the formulation of demand in a typical CGE model, we turn to the question of supply.
- We'll start with the basic cost minimization problem, and then turn to the problem of production in a competitive economy.
- In the process, we'll begin working with the CES function, which is widely used in constructing various components of CGE models.

Session Outline

1. The cost minimization problem
2. Example using CES
3. Building the model in GAMS
   - Setting up the model
   - Calibration
   - Simulation and testing

Cost Minimization

- Suppose that the firm uses inputs of labor (L) and capital (K), for which it must pay market prices w and r.
- Its technology is described by the production function $q = q(K, L)$. This function represents the relationship between inputs and the maximum output that can be produced, and is assumed to be continuous and to exhibit diminishing returns to each factor and CRTS.
- The firm seeks to minimize its expenditure for a given level of output, $\bar{q}$.

Formal Problem

The problem can be written:

$$\min \quad \mathscr{L} = rK + wL + \lambda[\bar{q} - q(K, L)]$$

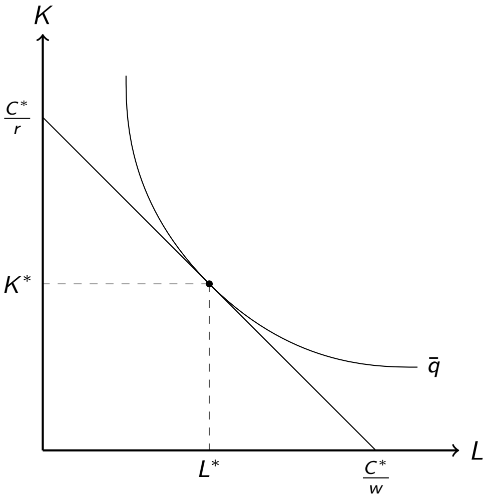
The first order conditions are:

$$\partial\mathscr{L}/\partial K = r - \lambda\partial q/\partial K = 0$$
$$\partial\mathscr{L}/\partial L = w - \lambda\partial q/\partial L = 0$$
$$\partial\mathscr{L}/\partial\lambda = \bar{q} - q(K, L) = 0$$

Interpretation

- The solution to the minimization problem is the simultaneous solution to these three equations for K, L and λ, expressed in terms of w, r and $\bar{q}$.
- At an optimum, each factor price is equal to the value of the marginal product of that factor.
- Solving explicitly for the optimal input bundles yields the firm's factor demand functions.

Geometric Interpretation



Example - CES Technology

Suppose the firm's technology can be described by the production function $q = \gamma[\delta K^\rho + (1-\delta)L^\rho]^{1/\rho}$, where $\rho \leq 1$ and $\rho \neq 0$. The factor demands will be:

$$K = \frac{\bar{q}\delta^\sigma r^{-\sigma}}{\gamma\left[(\delta r^{-\rho})^\sigma + ((1-\delta)w^{-\rho})^\sigma\right]^{\frac{1}{\rho}}}$$

$$L = \frac{\bar{q}(1-\delta)^\sigma w^{-\sigma}}{\gamma\left[(\delta r^{-\rho})^\sigma + ((1-\delta)w^{-\rho})^\sigma\right]^{\frac{1}{\rho}}}$$

where $\sigma = 1/(1-\rho)$ — the elasticity of substitution